Mathematics

7.10 The student recognizes that a physical or mathematical model can be used to describe the experimental and theoretical probability of real –life. The student is expected to

(B) Find the probability of independent events.

7.11 The Students understands that the way a set of data is displayed influences its interpretation. The student is expected to

(A) to select and use an appropriate representation for presenting and displaying relationships among collected data, including line plot, line graph, bar graph, stem and leaf plot, circle graph, and Venn diagrams, and justify the selection.

(B) make inferences and convincing arguments based on an analysis of given or collected data.

Technology Applications

The student demonstrates knowledge and appropriate use of hardware components, software programs, and their connections. The student is expected to

(1)(B) compare, contrast, and appropriately use the various input, processing, output, and primary/secondary storage devices.

(1)(C) demonstrate the ability to select and use software for a defined task according to quality, appropriateness, effectiveness, and efficiency.

The student uses appropriate computer-based productivity tools to create and modify solutions to problems. The student is expected to

(7)(H) use interactive virtual environments, appropriate to level, such as virtual reality or simulations.

Materials

Advances Preparation:

- Make the **Teacher Helper** transparency.

For each student:

- **The Helper Dilemma** activity sheet
- **The Choir Helper** activity sheet
- **Simulation** activity sheet
- TI-73 calculator

For each student group of students:

- Coin
- 10-sided number decahedron

For whole class demonstration:

- Transparency of **Teacher Helper**

ENGAGE

The Engage portion of the lesson is designed to create student interest in the concepts addressed. Technology is not used in this phase since the focus of this activity is to remind students of sample spaces and probability. This part of the lesson is designed for whole group instruction and groups of 2 students.

1. Display the **Teacher Helper** transparency on the overhead.
2. Read the problem as a class and ask students to take a moment to think about the problem on their own.
3. Prompt students to work with a partner to compile their thoughts and answer the questions.
4. Debrief using the facilitation questions.
5. Extend the discussion to find other probabilities such as the probability of getting an even number, the number 11, etc.

Facilitation Questions – Engage Phase

- How many students are in Mrs. Alexander's class?
  20
- How do you know?
  Answers may vary. This will hopefully lead into a discussion on sample spaces.
- What is a sample space?
  A sample space is the set of all possible outcomes for a given scenario.
- What is the sample space for this scenario?

Heads, 1	*Heads, 6*	*Tails, 1*	*Tails, 6*
Heads, 2	*Heads, 7*	*Tails, 2*	*Tails, 7*
Heads, 3	*Heads, 8*	*Tails, 3*	*Tails, 8*
Heads, 4	*Heads, 9*	*Tails, 4*	*Tails, 9*
Heads, 5	*Heads, 10*	*Tails, 5*	*Tails, 10*

- Are all of the possibilities equally likely? Why?
  Yes, there is only one head and one tail. Also, each number occurs only one time.
- What is the probability of the student assigned to Head, 6 being the helper?
  Have students refer back to the sample space. There is a $\frac{1}{20}$ *chance for the student with a Head, 6 to be the helper. Connect the sample space to finding the independent probability.*

EXPLORE

The Explore portion of the lesson provides the student with an opportunity to be actively involved in the exploration of the mathematical concepts addressed. This part of the lesson is designed for groups of 2 students.

1. Distribute a TI-73 to each student.
2. Distribute **The Helper Dilemma** activity sheet to each student.
3. The students should perform the experiment and record results.
4. Use the facilitation questions when students need help.

Facilitation Questions – Explore Phase

- How does the calculator help you generate the data?
  Answers may vary. The probability simulator performs the trials for you.
- How is using the calculator more beneficial than actually flipping the coin and rolling the number decahedron?
  Answers may vary. The calculator may be more reliable since it takes out the human error factor.
- How is using the calculator less beneficial than using the objects to simulate the experiment?
  Answers may vary. Lead students in a discussion that batches of calculators are programmed to start at the same random generating point. The data collected may be less random than data simulated with the actual objects.
- What do you know about the problem?
  Answers may vary. Have students verbalize the parts of the problem they know.
- What do you need to know?
  Answers may vary. Have students verbalize the parts of the problem they need to know through questioning.
- Have you worked problems like this before?
  Answers may vary. Relate the problem to prior learning and prior experiences.

EXPLAIN

The teacher directs the Explain portion of the lesson to allow the students to formalize their understanding of the TEKS addressed in the lesson.

1. Debrief **The Helper Dilemma**.
2. Discuss notation for probability of compound events (i.e. P(head, one)).

Facilitation Questions – Explain Phase

- What is the difference between theoretical and experimental probability?
  Answers may vary. Take this opportunity to review these topics.
- How were the experimental and theoretical probabilities the same?
  Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.
- How were the experimental and theoretical probabilities different?
  Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is helper so that when repeats occur, she knows to flip the coin and roll the number decahedron again.
- If the fractions were changed to percents, what would you expect the percents to total and why?
  Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.
- If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
  Answers may vary. Students may suggest that she use a number polyhedron with 12 sides. Some students may suggest that she flip a coin, roll the polyhedron and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.
- What can you conclude about the class where Mrs. Alexander assigned tails to girls and heads and prime numbers to boys?
  Answers may vary. Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.
- How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations?
  Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.
- In a previous question you were asked, "If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?" If 3 items were used to find the helper, for example, a coin, a number polyhedron and a spinner, how would the results be affected?
  Answers may vary. Discuss that a third item would increase outcomes.
- Do the items always have to yield equally likely results?
  Answers may vary. Discuss with students that they do not. Have students give examples where the outcomes are not equally likely.
- Are there times when the technology made the task easier? Why?
  Answers may vary. Some students may say that simulating the events using the calculator made the task easier.

Facilitation Questions – Explain Phase

- Are there times when the technology made the task more difficult? Why?
  Answers may vary. Some students may say recording the data into the lists and creating the graph would have been easier using only paper and pencil.

ELABORATE

The Elaborate portion of the lesson provides an opportunity for the student to apply the concepts of the TEKS to a new situation. This part of the lesson is designed for individual investigation.

1. Distribute a TI-73 calculator and **The Choir Helper** activity sheet to each student.
2. Have students should work individually to solve the problems using the calculator to help them.
3. Ask Facilitation Questions as needed.
4. Debrief by going over the worksheet.

Facilitation Questions – Elaborate Phase

- How do you use the calculator to help you?
  Possible answer: The calculator can be used to simulate the trials.
- How is this activity similar to **The Helper Dilemma**?
  Possible answer: Both activities used two items for simulation.
- How is this activity different?
  Possible answer: The sample spaces were different.
- How do you make predictions from results?
  Include a discussion here of scale factors. The simulation was for 50 trials, so to predict results for 100 trials use a scale factor of 2 (multiply by 2).
- How could you use a graph to show that the results for 50 trials, 100 trials, 250 trials, etc. are proportional?
  Answers may vary. Discuss with students that a graph could be made charting each individual outcome. For example, chart the results for P(tail, letter A) for 50 trials by letting the List 1 represent trials and the List 2 represent outcomes. Other outcomes for the same probability could be entered into the lists and then graphed. A discussion of the data points should follow. The points should appear to be in a straight line that would travel through the origin. Thus, the data is proportional.

EVALUATE

The Evaluate portion of the lesson provides the student with an opportunity to demonstrate his or her understanding of the TEKS addressed in the lesson.

1. Distribute **Simulation** activity sheet to each student.
2. Upon completion of **Simulation** activity sheet, the teacher should use a rubric to assess student understanding of the concepts addresses in this lesson.

Answers and Error Analysis for selected response questions:

Question Number	TEKS	Correct Answer	Conceptual Error	Conceptual Error	**Procedural** Error	Procedural Error	**Guess**
1	7.10B	D	A		C		B
2	7.11B	B	A	D			C
3	7.11A	A	B	C			D
4	7.10B	A	D		B	C	

The Helper Dilemma – (Possible Answers)

1. Use two TI-73 calculators to simulate the probability. Each student should hold a calculator. One calculator should be used to simulate the coin toss and the other used to simulate rolling a 10-sided number decahedron. Follow the instructions below to simulate the events with the calculators. Combine the results and place a tally mark in the frequency table below. Perform the experiment 40 times.

Coin Toss
APPS 7: Prob Sim Press any key 2. Toss Coin Toss (Window)

Roll Dice
APPS 7: Prob Sim Press any key 1. Roll Dice Set (Zoom) Sides: 10 OK (Graph) Roll (Window)

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

(continue: The Helper Dilemma)

2. Graph the data using the instructions below. Sketch the resulting graph.

Graph Data	Sketch graph here.
Enter the frequency data in L2 of the LIST feature. 2nd Plot (Y=) 1: Plot 1 On Type: Pie Chart Graph	*Graph will vary, but should match results in table.*

3. Find the experimental probability for each.

Answers may vary experiment by experiment.

Combination	Experimental Probability
Head, 1	
Head, 2	
Head, 3	
Head, 4	
Head, 5	
Head, 6	
Head, 7	
Head, 8	
Head, 9	
Head, 10	

Combination	Experimental Probability
Tail, 1	
Tail, 2	
Tail, 3	
Tail, 4	
Tail, 5	
Tail, 6	
Tail, 7	
Tail, 8	
Tail, 9	
Tail, 10	

(continue: The Helper Dilemma)

4. How were the experimental and theoretical probabilities the same? Explain.
   Answers may vary. Depending on the experiment, some may say that the experimental probabilities were close to being equally distributed.

5. How were the experimental and theoretical probabilities different? Explain.
   Answers may vary. Depending on the experiment, some of the combinations may have occurred more than others. Possibly discuss at this time how Mrs. Alexander should keep track of who is helper so that when repeats occur, she knows to flip the coin and roll the decahedron again.

6. If the fractions were changed to percents, what would you expect the percents to total and why?
   Answers may vary. Lead students to the understanding that the experiment is a whole event, so that the percents would add to 100% and the fractions to 1 whole.

7. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?
   Answers may vary. Students may suggest that she use a number polyhedron with 12 sides. Some students may suggest that she flip a coin, roll the polyhedron and use a spinner with 3 or 4 sections. Some students may suggest that she use a deck of cards and assign each student a card from the deck.

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

8. What is the sample space for this class?

Head, 1	*Head, 6*	*Tail, 1*	*Tail, 6*
Head, 2	*Head, 7*	*Tail, 2*	*Tail, 7*
Head, 3	*Head, 8*	*Tail, 3*	*Tail, 8*
Head, 4	*Head, 9*	*Tail, 4*	*Tail, 9*
Head, 5	*Head, 10*	*Tail, 5*	*Tail, 10*

9. What can you conclude about this particular class? Explain.
   Not all of the combinations in the sample space will be used for this class. This class has more girls than boys since more combinations are assigned to girls than boys.

10. Which gender is most likely to be the helper? Explain.
    A girl is most likely to be the helper since more combinations are assigned to girls than boys.

(continue: The Helper Dilemma)

11. What is the probability of a girl being the helper? Explain.

There is a $\frac{1}{2}$ chance of getting a tail and a $\frac{10}{10}$ chance of getting a number on the decahedron. Combine the probabilities using multiplication, $\frac{1}{2} \cdot \frac{10}{10}$, to get a $\frac{10}{20} = \frac{1}{2}$ chance of getting a girl helper.

12. What is the probability of a boy being the helper? Explain.

There is a $\frac{1}{2}$ chance of getting a head and a $\frac{4}{10}$ chance of getting a prime number on the decahedron. Combine the probabilities using multiplication, $\frac{1}{2} \cdot \frac{4}{10}$, to get a $\frac{4}{20} = \frac{1}{5}$ chance of getting a boy helper.

13. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

Answers may vary. Mrs. Alexander could use the coin and a bag of marbles with 4 different colors for the boys or a spinner with 4 equal sections.

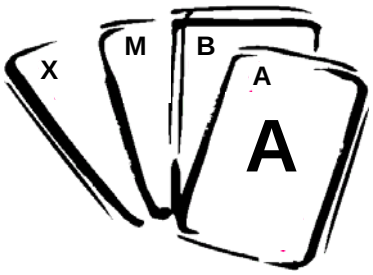
The Choir Helper – (Possible Answers)

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code," Mr. Roberts cannot use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

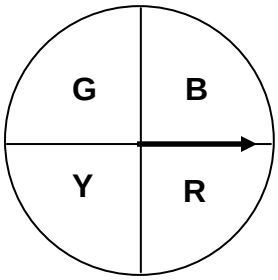
A Coin

A Six-Sided Number Cube

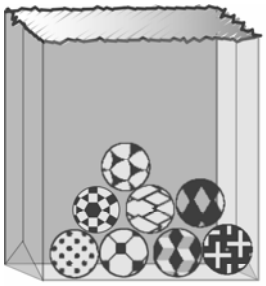
A 10-sided Number Decahedron with the numbers 1-10



A Set of Alphabet Cards A-Z

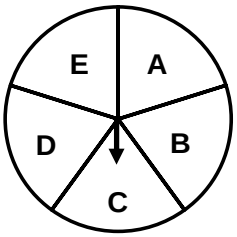


A Spinner



A Bag of 8 Different Marbles

A 12-sided Number Dodecahedron with the numbers 1-12



A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

   Pair 1: A bag of 8 marbles and the 6-sided number cube (48 assignments)

   Pair 2: The coin and set of Alphabet Cards (52 assignments)

   Pair 3: The spinner of colors and the 12-sided number dodecahedron (48 assignments)

   Pair 4: The spinner with letters and the 10-sided number decahedron (50 assignments)

2. Choose one of the pairs of items above and describe how to simulate the event using the calculator.

   Answers may vary experiment to experiment.

3. Use the plan outlined in #2 to simulate the event for 50 trials. Create a table to record the results.

   Answers may vary experiment to experiment.

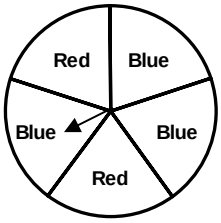
4. From the above results, predict the results if the event had been simulated for 100 trials.

   Answers may vary, but should include that the results in #3 should be multiplied by a scale factor of 2.

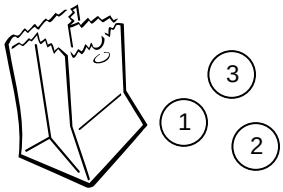
Simulation – (Possible Answers)

Use the following items to simulate an experiment.

A Coin



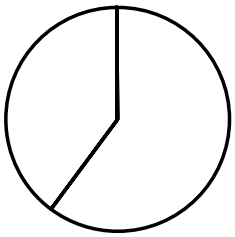
A Spinner



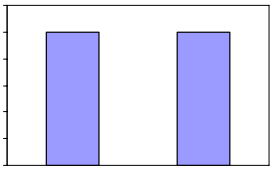
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represents the results of the experiment? Justify your reasoning.

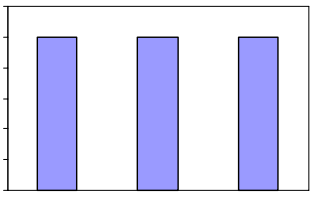
A.



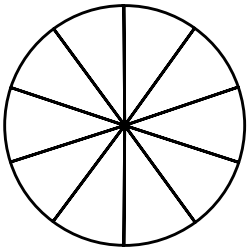
C.



B.



D.

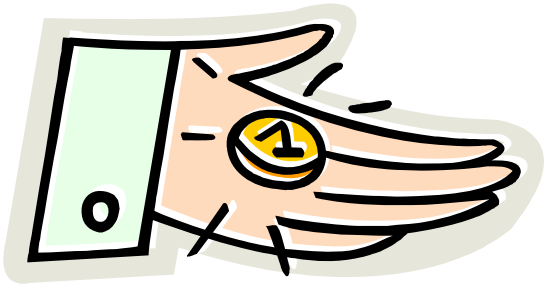


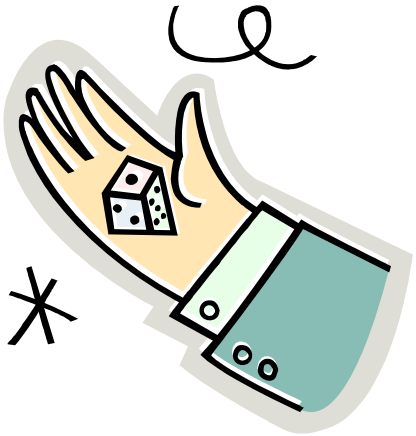
Answer: The graph in A best represents the experiment. In the experiment, the spinner has more blue than red. A circle graph representing the results of blue to red would show a larger section for blue.

Teacher Helper

Mrs. Alexander assigns the helping job in her class by flipping a coin and rolling a 10-sided number decahedron. Each student in her class is assigned a combination of a head or tail and a number from the decahedron. Students in the same class do not share the same combination.

- If all the possible combinations are assigned, how many students are in Mrs. Alexander's class?
- What are the possible combinations?





The Helper Dilemma

1. Use two TI-73 calculators to simulate the probability. Each student should hold a calculator. One calculator should be used to simulate the coin toss and the other used to simulate rolling a 10-sided number decahedron. Follow the instructions below to simulate the events with the calculators. Combine the results and place a tally mark in the frequency table below. Perform the experiment 40 times.

Coin Toss
APPS 7: Prob Sim Press any key 4. Toss Coin Toss (Window)

Roll Dice
APPS 7: Prob Sim Press any key 3. Roll Dice Set (Zoom) Sides: 10 OK (Graph) Roll (Window)

Combination	Tally	Frequency
Head, 1		
Head, 2		
Head, 3		
Head, 4		
Head, 5		
Head, 6		
Head, 7		
Head, 8		
Head, 9		
Head, 10		

Combination	Tally	Frequency
Tail, 1		
Tail, 2		
Tail, 3		
Tail, 4		
Tail, 5		
Tail, 6		
Tail, 7		
Tail, 8		
Tail, 9		
Tail, 10		

2. Graph the data using the instructions below. Sketch the resulting graph.

Graph Data

Enter the frequency data in L2 of the LIST feature.

2nd
Plot (Y=)
1: Plot 1
On
Type: Pie Chart
Graph

Sketch graph here.

3. Find the experimental probability for each.

Combination	Experimental Probability
Head, 1	
Head, 2	
Head, 3	
Head, 4	
Head, 5	
Head, 6	
Head, 7	
Head, 8	
Head, 9	
Head, 10	

Combination	Experimental Probability
Tail, 1	
Tail, 2	
Tail, 3	
Tail, 4	
Tail, 5	
Tail, 6	
Tail, 7	
Tail, 8	
Tail, 9	
Tail, 10	

(continue: The Helper Dilemma)

4. How were the experimental and theoretical probabilities the same? Explain.

5. How were the experimental and theoretical probabilities different? Explain.

6. If the fractions were changed to percents, what would you expect the percents to total and why?

7. If Mrs. Alexander has more students enrolled in her class, how can she change or modify her procedure for finding a helper?

Use the following information to answer questions 8-13.

In one particular class, Mrs. Alexander assigned combinations with Heads and a prime number to only boys and combinations with Tails to only girls.

8. What is the sample space for this class?

9. What can you conclude about this particular class? Explain.

10. Which gender is most likely to be the helper? Explain.

11. What is the probability of a girl being the helper? Explain.

(continue: The Helper Dilemma)

12. What is the probability of a boy being the helper? Explain.

13. How could Mrs. Alexander change or modify her procedure for finding a helper in this class to eliminate the extra combinations? Explain.

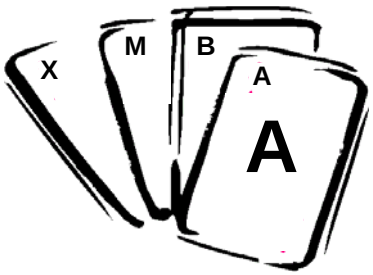
The Choir Helper

The choir teacher, Mr. Roberts, heard Mrs. Alexander in the teacher's lounge describe her method for assigning a helper. He thought the idea would be a big help in his classes. Since his choir classes sometimes have between 45 and 50 students and no students can be assigned the same "code," Mr. Roberts cannot use the coin and 10-sided number decahedron. Mrs. Alexander gave Mr. Roberts 8 different items that he could use to assign helpers in his class.

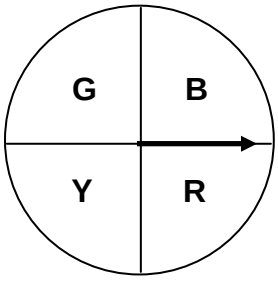
A Coin

A Six-Sided Number Cube

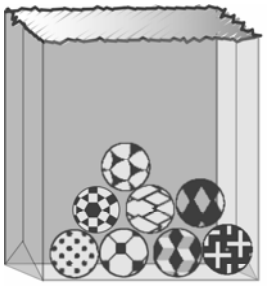
A 10-sided Number Decahedron with the numbers 1-10



A Set of Alphabet Cards A-Z

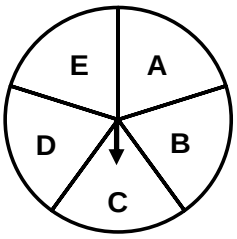


A Spinner



A Bag of 8 Different Marbles

A 12-sided Number Dodecahedron with the numbers 1-12



A Spinner

(continue: The Choir Helper)

1. Help Mr. Roberts pair the items together that he can use to assign helpers. There will be 4 pairs. Justify your reasoning for each pair made and tell how many assignments for helpers could be made from each pair.

2. Choose one of the pairs of items above and describe how to simulate the event using the calculator.

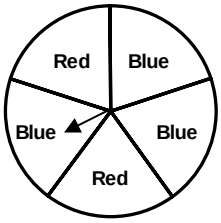
3. Use the plan outlined in #2 to simulate the event for 50 trials. Create a table to record the results.

4. From the above results, predict the results if the event had been simulated for 100 trials.

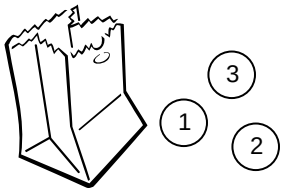
Simulation

The following items are being used to simulate an experiment.

A Coin



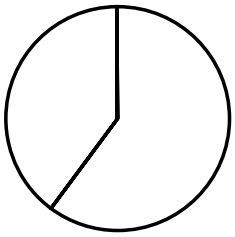
A Spinner



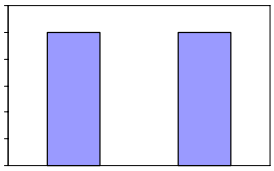
A Bag of 3 Marbles
Numbered 1-3

Which of the following graphs best represent the results of the experiment? Justify your reasoning

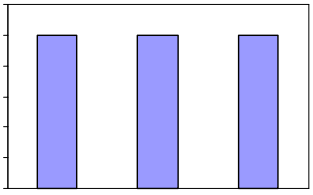
A.



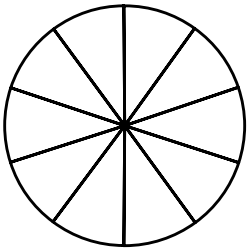
C.



B.



D.



1. Corbyn has a standard code of dress at his school. He can wear a white or green shirt with navy or khaki pants. He had 3 white shirts and 2 green shirts in his shirt drawer and 1 pair of navy pants and 3 pairs of khaki pants in his pants drawer. What is the probability that Corbyn will reach in both drawers, without looking, and get a white shirt and navy pants?

   A $\frac{17}{20}$

   B $\frac{4}{9}$

   C $\frac{3}{25}$

   D $\frac{3}{20}$

2. A 6-sided number cube, a spinner divided into 3 equal parts labeled A, A, B, and a coin are used for an experiment. Ozzie calculated the theoretical probability of an event where the number cube was rolled, coin tossed, and spinner spun. His calculation was $\frac{1}{3}\cdot\frac{1}{2}\cdot\frac{2}{3}=\frac{2}{18}=\frac{1}{9}$. For which of the following events did Ozzie calculate the probability?

   A P(even number, head, B)

   B P(1 or 2, head, A)

   C P(prime number, tail, A)

   D P(odd number, tail, A)

3. The letters of the word WINNER are cut apart and placed in a bag. A letter was drawn from the bag and a coin tossed at the same time. Results were recorded and the letter was placed back into the bag. Which of the following could NOT be used to represent the experimental data?

   A Venn diagram

   B Bar graph

   C Circle graph

   D Line Plot

4. A container of markers containing 3 red, 1 yellow, 2 green and 4 blue are placed at the map center in social studies. The rule is you can only use one marker at a time so that everyone will have a marker to use. What is the probability of reaching into the container without looking for each use and getting a red marker, a blue marker and then a yellow marker?

   A $\frac{3}{250}$

   B $\frac{12}{30}$

   C $\frac{12}{100}$

   D $\frac{8}{10}$